

Short-range attractive Hubbard models: current applications to superconductivity and beyond

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The attractive Hubbard model and its extensions provide one of the fundamental theoretical frameworks for describing superconductivity in systems with short coherence lengths and strong pairing correlations [1–4]. Over the past several decades, this model and its numerous generalizations have played a central role in the study of high-temperature superconductivity, charge ordering, pseudogap phenomena, collective excitations, and strongly correlated quantum matter, e.g., [1–7]. Beyond its fundamental importance, the physics described by these models is relevant to a broad range of systems, including cuprate and barium bismuthate superconductors, ultra-cold atomic gases in optical lattices, layered organic conductors, and transition-metal dichalcogenides. These models can also describe charge-ordered phases, particularly in two-dimensional systems where the competition and coexistence of different collective orders play a central role [8, 9]. Such phenomena have gained renewed attention in the context of atomically thin materials, moiré superlattices, and van der Waals heterostructures, where superconductivity frequently emerges in close proximity to charge-ordered, insulating, or other correlated electronic states. Understanding the mechanisms governing superconductivity in such systems is essential not only for advancing condensed-matter theory but also for the development of future quantum technologies, e.g., [10, 11].

In this presentation, I will try to discuss the fundamental concepts underlying the description of superconductivity in narrow-band systems with local, non-retarded attractive interactions and trace their evolution from early mean-field approaches to modern many-body theories. Particular attention will be paid to the BCS-BEC crossover, the role of strong correlations and fluctuation effects, charge ordering and phase-separation phenomena, and recent advances in our understanding of superconductivity in strongly interacting quantum materials.

References

- [1] R. Micnas, J. Ranninger, S. Robaszkiewicz, *Rev. Mod. Phys.* 62, 113 (1990).
- [2] S. Robaszkiewicz, R. Micnas, K. A. Chao, *Phys. Rev. B* 23, 1447 (1981).
- [3] R. Micnas, J. Ranninger, S. Robaszkiewicz, *J. Phys. C* 21, L145 (1988).
- [4] R. Micnas *et al.*, *Phys. Rev. B* 36, 9410 (1988).
- [5] J. Spałek *et al.*, *Phys. Rep.* 959, 1 (2022).
- [6] W. R. Czart, K. J. Kapcia, R. Micnas, S. Robaszkiewicz, *Physica A* 585, 126403 (2022).
- [7] A. Cichy, K. J. Kapcia, A. Ptok, *Phys. Rev. B* 105, 214510 (2022).
- [8] A. Alekseev, A. Cichy, K. J. Kapcia, *Phys. Rev. B* 112, 115155 (2025).
- [9] Y. Zambrano *et al.*, *Phys. Rev. B* 114, 065104 (2026).
- [10] A. Hamo *et al.*, *Nature* 535, 395 (2016).
- [11] S. I. Uchida, *J. Phys. Soc. Jpn.* 90, 111001 (2021).